REFERENCE TEMPERATURE IN TWO-DIMENSIONAL PROBLEMS OF HEAT CONDUCTION

A. V. Bulyga and V. V. Kharitonov

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The problem of determining the average temperature in calculating the heat transfer between coaxial cylinders is considered. It is shown that the true value of the average temperature depends on the geometry of the heat transfer surfaces and is less than the arithmetic mean value usually employed.

In analyzing and generalizing the results of heat transfer experiments it is usual to refer the data obtained to a certain average temperature characteristic of the process.

In the case of heat transfer between a fluid moving through a channel and the channel walls [1] it is proposed to average the temperature with respect to the enthalpy of the fluid:

$$t_{av}=\frac{\int\limits_f \rho c_p v t df}{\int\limits_f \rho c_p v df}. \quad (1)$$

When ρ = const and c_p = const this formula becomes

$$t_{av}=\frac{1}{V}\int\limits_f v t df, \quad (2)$$

i.e., the averaging parameter is the volume flow rate.

However, if the velocity is the same over the channel cross section or equal to zero (case of pure heat conduction), the averaging can be performed with respect to the channel cross section and the averaging formula becomes

$$t_{av}=\frac{1}{f}\int\limits_f t df. \quad (3)$$

At present, as the reference temperature for both one-dimensional and two-dimensional problems it is customary to take either the temperature of the initial flow (or ambient medium) or the arithmetic mean

$$t_{av\cdot a}=\frac{t_1+t_2}{2}=t_2+\frac{t_1-t_2}{2}=t_2+\frac{\Delta t}{2}. \quad (4)$$

However, it appears that the latter expression, which is valid for the case of heat transfer by pure conduction between plates of infinite length, is only approximately correct for two-dimensional problems (for example, for heat transfer between two coaxial cylinders) and only if the thermophysical characteristics of the medium do not depend too strongly on temperature.

The exact value of the average temperature for the simplest case of the two-dimensional problem—heat transfer by condition between two coaxial cylinders of infinite length—can be obtained as follows.

Since the cross-sectional area normal to the axis of a coaxial cylinder can be expressed as $f=\pi r^2$, whence $df = 2\pi r dr$, while the temperature is expressed by the equation [1]

$$t=t(r)=t_1-\frac{q}{2\pi\lambda}\ln\frac{r}{r_1}, \quad (5)$$

for the case in question Eq. (3) becomes

$$t_{av}=\frac{2}{r_2^2-r_1^2}\int\limits_{r_1}^{r_2}\left(t_1-\frac{q}{2\pi\lambda}\ln\frac{r}{r_1}\right) r dr. \quad (6)$$

After integration and algebraic transformations we obtain

$$t_{av}=t_1-\frac{q}{2\pi\lambda}\left(\frac{\ln\frac{r_2}{r_1}}{1-(r_1/r_2)^2}-\frac{1}{2}\right). \quad (7)$$

Since

$$q=\frac{2\pi\lambda}{\ln\frac{r_2}{r_1}}(t_1-t_2), \quad (8)$$

after substituting (8) into (7) we obtain

$$t_{av}=t_1-(t_1-t_2)\left[\frac{1}{1-(r_1/r_2)^2}-\frac{1}{2\ln\frac{r_2}{r_1}}\right] \quad (9)$$

or after transformations

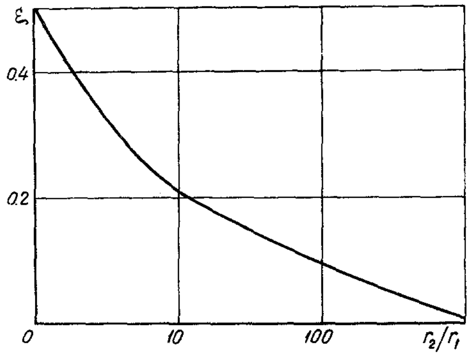
$$t_{av}=t_2+\xi\Delta t, \quad (10)$$

where

$$\xi=\frac{1}{2}\left\{2-\left[\frac{2}{1-(r_1/r_2)^2}-\frac{1}{\ln\frac{r_2}{r_1}}\right]\right\}. \quad (11)$$

An analysis of (11) shows that when r_2/r_1 varies from 1 to infinity the correction ξ varies from 0.5 to 0 (figure).

To illustrate the possible errors which may be introduced in calculating the reference temperature from Eq. (4), we offer the following example. If we calculate the reference temperature t'_{av} from Eq. (4), then in the experimental determination of the thermal conductivity λ by the hot-wire method [2] (r_2/r_1 = 10.7, Δt = 100°, t_2 = 20° C) instead of the true temperature,



The correction ξ as a function of the ratio r_2/r_1.

given by Eq. (10), $t_{av} = 40.5°$ C ($T_{av} = 313.7°$ K) we will have $t'_{av} = 70°$ C ($T'_{av} = 343.2°$ K).

The temperature dependence of the thermal conductivity of gases is expressed by the formula [2]

$$\lambda = \lambda_0 \left(\frac{T}{T_0}\right)^n, \quad (12)$$

where n = 1.53 for ammonia and n = 2.03 for nitrogen.

Calculating the thermal conductivities, we obtain $\lambda_{am} = 1.23\lambda_{0am}$, $\lambda_{ni} = 1.32\lambda_{0ni}$ when $T_{av} = 313.7°$ K and $\lambda'_{am} = 1.42\lambda_{0am}$, $\lambda'_{ni} = 1.58\lambda_{0ni}$ when $T'_{av} = 343.2°$ K.

NOTATION

t and T denote temperature; v is the velocity; c_p is the specific heat at constant pressure; ρ is the density; V is the volume flow rate; r is the radius; q is the specific heat flux; λ is the thermal conductivity. Subscripts: 1—inner (wall); 2—outer (medium); av—average, am—ammonia, ni—nitrogen.

REFERENCES

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Institute of Heat and Mass Transfer AS BSSR, Minsk